



A DOA matrix method for two-dimensional angle estimation using cylindrical hydrophone array

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ABSTRACT

Cylindrical hydrophone arrays (CHAs) enable full-space two-dimensional direction of arrival (DOA) estimation for underwater targets. However, conventional methods face an inherent trade-off between computational complexity and estimation accuracy. To address this challenge, this paper proposes an improved DOA matrix method that decouples elevation and azimuth estimation via the axial translational invariance of the CHA and solves the azimuth angles using the Gauss-Newton iterative method. A complete metrological framework, encompassing bias, root mean square error (RMSE), resolution and expanded uncertainty, is established to characterize the measurement performance. Simulations and anechoic tank experiments demonstrate that the proposed method achieves elevation accuracy comparable to MUSIC and concedes only a modest margin in azimuth accuracy, while reducing computational complexity by up to two orders of magnitude with a 1° grid. In addition, it significantly outperforms TLS-ESPRIT in azimuth accuracy, while matching it in elevation accuracy, with only a modest latency increase. Numerical results further indicate improved finite-snapshot resolution and enhanced robustness to coherent sources. Millisecond-order latency and azimuth RMSE below 3° are verified for both stationary and moving sources, with elevation RMSE below 1° for moving sources. This method provides an effective solution for real-time, high-precision 2D DOA measurement using CHAs.

1. Introduction

Underwater target direction of arrival (DOA) estimation is fundamentally a measurement problem of two physical quantities: the azimuth angle and the elevation angle of an acoustic source. It has important application value in various fields, including national defense [1], marine resource exploration [2] and environmental monitoring [3]. Hydrophone arrays are a critical technology for underwater target DOA estimation [4–6]. The fundamental principle relies on utilizing the time delay or phase differences among signals received by individual array elements to estimate the target DOA. Conventional arrays, such as linear and planar arrays, have been widely adopted due to their straightforward structures and mature processing algorithms [7,8]. Their physical configurations, however, possess inherent limitations. Linear arrays, while featuring the simplest geometry, are confined to one-dimensional direction finding and suffer from the left–right ambiguity [9]. Although vector hydrophone arrays can overcome this ambiguity through pressure–velocity measurements [10], they are still linear arrays and thus cannot provide full 2D estimation. Planar arrays enable two-

dimensional (2D) DOA estimation by providing both azimuth and elevation information, which resolves the left–right ambiguity inherent in linear arrays [11]. However, as they are typically deployed in either horizontal or vertical orientations, their detection coverage is confined to a half-space region in the broadside direction. Consequently, unambiguous DOA estimation across the full spatial domain cannot be achieved. Cylindrical hydrophone arrays (CHAs) represent a significant development direction in underwater acoustic detection technology [12]. Owing to their circumferential symmetry and vertical aperture structure, they enable unambiguous 2D DOA estimation over the full spatial domain.

However, the complex geometry of CHAs introduces distinct challenges for DOA estimation. First, the array manifold of such an array lacks the Vandermonde structure characteristic of uniform linear arrays. Consequently, the phase component within the steering vector exhibits a nonlinear dependence on the angular parameters, precluding a direct analytical solution [13]. Secondly, to achieve adequate angular resolution, a cylindrical array typically necessitates a dense distribution of elements along both its circumference and axis. This configuration leads

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to a high-dimensional covariance matrix, thereby causing a substantial increase in computational complexity. Furthermore, the 2D DOA estimation for a cylindrical array entails the coupled estimation of both azimuth and elevation angles. This parameter coupling significantly complicates the estimation problem. Employing a 2D spectral peak search method to solve for these angles results in a computational burden that grows exponentially with the required angular search range and resolution [14]. These challenges, inherent to the cylindrical array's geometry, impose stringent demands on both the performance and computational efficiency of DOA estimation algorithms.

Although numerous mature methods exist for DOA estimation, their practical applications are often limited by inherent drawbacks. Among them, subspace-based methods represent a major category, including Multiple Signal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT). MUSIC [15] exploits the orthogonality between the signal and noise subspaces to construct a spatial spectrum function, providing high resolution and applicability to arbitrary array geometries. However, its application to 2D DOA estimation with CHAs still require a 2D spectral peak search, leading to high computational complexity. In contrast, ESPRIT [16] leverages the translational invariance structure of an array to achieve closed-form DOA estimation via rotational invariance, thereby circumventing the need for spectral peak search. A fundamental limitation of conventional ESPRIT is its requirement for a specific translational invariance structure, which precludes its direct application to uniform circular arrays (UCAs) or CHAs. Given that a UCA serves as the fundamental building block of a CHA, improved methods developed for UCAs are highly relevant to CHA DOA estimation. A prominent example is UCA-ESPRIT, which employs phase mode excitation to transform a UCA into a virtual uniform linear array, thereby achieving closed-form estimation in the beamspace domain [17,18]. Another variant, Total Least Squares ESPRIT (TLS-ESPRIT), improves estimation accuracy by solving the rotational invariance equation under the total least squares criterion, and this criterion accounts for perturbations in both subarray subspaces [19]. A common drawback of these methods is that, to satisfy the translational invariance requirement, only a subset of array elements can be utilized to form subarrays at any given time. This constraint results in an effective aperture that is smaller than the physical aperture, ultimately limiting their multi-target resolution capability.

Another important class of 2D DOA estimation methods is the so-called DOA matrix method, originally introduced by Yin et al. [20]. Its core principle relies on exploiting the translational invariance in the array geometry. Specifically, it constructs a distinct matrix, referred to as the DOA matrix, by using the auto-correlation and cross-correlation information derived from this translational invariance. Performing eigenvalue decomposition on this specific matrix enables the simultaneous estimation of both azimuth and elevation angles with automatic parameter pairing. In recent years, this method has been further developed in terms of performance and applicability. For instance, Dai et al. [21] enhanced estimation accuracy for double parallel uniform linear arrays by constructing an extended DOA matrix that better utilizes auto-correlation and cross-correlation information. Li et al. [22] integrated this method with parallel nested linear arrays, using their difference coarray properties to increase the degrees of freedom, thus enabling the detection of more sources. Chen et al. [23] applied an extended DOA matrix method to a model comprising a single linear array with time-delay units, achieving joint estimation of the azimuth and frequency, which is also applicable to non-uniform linear arrays. The DOA matrix method has also been adapted for UCAs. Wang et al. [24] constructed a polarization-DOA matrix for a UCA of electromagnetic vector sensors. By recovering the rank of the covariance matrix through axial virtual translation, they achieved joint estimation of

direction and polarization for coherent sources with automatic parameter pairing. Within this DOA matrix formulation, the elevation angle follows directly from the axial translational invariance, whereas the azimuth is derived from the circular aperture. Since the phase of the UCA steering vector depends nonlinearly on the element index, its manifold lacks the Vandermonde structure required for polynomial rooting. Consequently, estimating the azimuth via Root-MUSIC requires a preliminary beamspace transformation that projects the signal into a beamspace virtual uniform linear array [17]. However, this transformation introduces phase mode truncation errors, which become more pronounced as the number of array elements decreases [25]. This limitation is inherent to Root-MUSIC variants for circular or cylindrical arrays.

In addition to the above matrix-based approaches, tensor decomposition methods have attracted growing attention for multi-dimensional DOA estimation. By modeling received data as a higher-order tensor, these methods preserve the inherent multilinear structure among spatial and temporal dimensions, achieving superior estimation performance. Representative works include a tensor-train decomposition framework for joint 2D direction of departure (DOD) and 2D-DOA estimation in bistatic MIMO radar with uniform planar arrays [26] and a covariance tensor subspace algorithm via higher-order singular value decomposition for DOD and DOA estimation of non-line-of-sight targets in RIS-aided bistatic MIMO radar [27]. Nevertheless, these methods are developed for bistatic MIMO radar systems and rely on array geometries and signal models that differ fundamentally from those of CHAs. Consequently, their direct extension to CHA-based DOA estimation, where the steering vector lacks Vandermonde structure, remains an open and non-trivial problem.

In parallel, DOA estimation methods based on sparse reconstruction have gained considerable attention [28,29]. These methods reformulate the DOA estimation problem as a sparse signal reconstruction task, offering advantages such as single-snapshot estimation capability and high resolution. Based on their dependence on angular grid discretization, sparse reconstruction approaches are categorized into on-grid methods and off-grid methods. On-grid methods require predefined discretization of the angular space. This discretization introduces grid mismatch error when the true DOA lies off the grid points. Conversely, increasing the grid density to mitigate this error substantially increases the dimension of the dictionary matrix and the computational burden [30]. Off-grid methods, while avoiding the grid mismatch issue, face a different constraint. Most of these techniques necessitate measurement data from arrays with a regular sampling structure, such as uniform linear arrays. This requirement hinders their direct application to more complex geometries like cylindrical arrays [31]. In addition, deconvolution-based sparse methods have been explored for joint TOA-DOA estimation of wideband underwater multipath signals, where sparse optimization algorithms are applied as post-processors to the cross-correlation-based beamforming output to suppress the main-lobe width and side-lobe level, thereby achieving high-resolution estimation for coherent signals [32].

In summary, existing methods for CHA-based 2D DOA measurement face an unresolved trade-off between measurement accuracy and latency (real-time capability). MUSIC achieves high angular resolution, but its requirement for a 2D spectral peak search incurs prohibitive computational cost. ESPRIT-type and the DOA matrix methods provide closed-form solutions, but are inherently dependent on the array's translational invariance, complicating their direct adaptation to cylindrical arrays. Furthermore, improved versions that employ beamspace transformation often suffer from induced model errors and performance degradation with a limited number of elements. Tensor decomposition methods excel in MIMO radar contexts but do not directly extend to CHA

geometries. Sparse reconstruction methods offer super-resolution but are hampered by high computational complexity and sensitivity to grid precision. Although FFT-accelerated sparse deconvolution methods exploiting the shift-invariant property of uniform linear arrays, such as FFT-FISTA and FFT-ADMM, have been developed to improve real-time DOA estimation performance [33], their adaptation to the complex geometry of CHAs remains non-trivial. Therefore, developing a DOA estimation method for CHAs that effectively balances accuracy with efficiency carries significant theoretical importance and practical engineering value.

To tackle the aforementioned challenges, this paper proposes a novel DOA matrix measurement method specifically tailored for CHAs. The contributions of this work are twofold:

- (1) A new elevation and azimuth measurement method via extending the DOA matrix framework from parallel or nested linear arrays and uniform circular arrays to 3D CHAs. A DOA matrix is constructed by exploiting the axial translational invariance inherent in a cylindrical array. Performing eigenvalue decomposition on this matrix yields estimates of elevation angles from the larger eigenvalues, while the corresponding eigenvectors provide estimates for azimuth angles. This process naturally decouples the estimation of the two angular parameters, completely eliminating the need for 2D spectral search.
- (2) A dedicated solver for nonlinear azimuth measurement in the new DOA matrix framework: The proposed method directly confronts the nonlinear relationship between the phase of a UCA steering vector and the azimuth angle. The azimuth estimation is formulated as a nonlinear least squares problem and solved efficiently using a two-stage approach: a coarse grid search for initialization, followed by Gauss-Newton iteration. This solver converges in a few iterations, reduces computation and maintains accuracy close to the Cramér-Rao bound. In contrast to existing UCA-based methods that rely on beamspace transformation, the proposed method avoids the associated phase-mode truncation bias, thereby resolving the inherent conflict between measurement accuracy and real-time capability.

To provide a complete metrological characterization, this paper establishes an uncertainty propagation model, quantifies bias, repeatability, and resolution, while presenting a full uncertainty budget at a representative broadside position based on controlled anechoic tank experiments. The remainder of this paper is organized as follows. Section 2 elaborates on the proposed DOA measurement method, including the signal model, the specific implementation steps and the measurement uncertainty analysis. Section 3 presents a systematic metrological evaluation and validation via numerical simulations. Section 4 specifies the configuration and experimental procedure of the

anechoic tank experiments. Section 5 discusses the experimental results, including the uncertainty budget and the method's limitations in complex environments. Finally, Section 6 draws conclusions and summarizes the whole paper.

2. Methods

2.1. Received signal model of CHA

The geometry of the CHA is shown in Fig. 1. A coordinate system is established with its origin at the center of the array's base, which lies in the xOy plane. Consider an N -layer CHA with a fixed inter-layer vertical spacing of h . Each layer consists of M elements uniformly distributed on a circle with radius r . The first element of each layer is positioned in the xOz plane. The DOA of a source signal is described by an azimuth angle θ and an elevation angle φ . The azimuth angle $\theta \in [0, 2\pi]$ is defined as the angle between the projection of the signal's incident direction onto the xOy plane and the positive x -axis. The elevation angle $\varphi \in [0, \pi]$ is the angle between the incident direction and the positive z -axis. Based on this geometry, the position vector of the m -th element of the n -th layer can be expressed as $\mathbf{p}_{nm} = (r\cos(\gamma_m), r\sin(\gamma_m), (n-1)h)$, where $\gamma_m = 2\pi m/M$, $m = 0, 1, \dots, M-1$.

Consider a far-field narrowband source with wavelength λ . Its signal impinges on the array from the direction given by the unit vector $\mathbf{u} = (\sin \varphi \cos \theta, \sin \varphi \sin \theta, \cos \varphi)$. At a given time instant, the relative signal delay at the m -th element of the n -th layer relative to the origin is:

$$\tau_{nm} = \mathbf{p}_{nm} \mathbf{u} / c = (r \sin \varphi \cos(\theta - \gamma_m) + (n-1)h \cos \varphi) / c \quad (1)$$

where c denotes the speed of sound. Consequently, the steering vector of the CHA is derived as:

$$\mathbf{a}(\theta, \varphi) = \begin{bmatrix} \exp(-j\omega_0 \tau_{11}) \\ \exp(-j\omega_0 \tau_{12}) \\ \vdots \\ \exp(-j\omega_0 \tau_{NM}) \end{bmatrix} = \begin{bmatrix} \exp(-jk_0 r \sin \varphi \cos(\theta - \gamma_1)) \\ \exp(-jk_0 r \sin \varphi \cos(\theta - \gamma_2)) \\ \vdots \\ \exp(-jk_0 (N-1)h \cos \varphi) \exp(-jk_0 r \sin \varphi \cos(\theta - \gamma_M)) \end{bmatrix} \quad (2)$$

where $k_0 = 2\pi/\lambda$ is the wavenumber and ω_0 is the angular frequency.

Assuming K such far-field narrowband sources ($K < MN$) impinge on the array. At time t , the signal received by the m -th element of the n -th layer can be expressed as:

$$x_{nm}(t) = \sum_{i=1}^K \mathbf{a}_{nm}(\theta_i, \varphi_i) s_i(t) + n_{nm}(t), i = 1, 2, \dots, K \quad (3)$$

where $\mathbf{a}_{nm}(\theta_i, \varphi_i)$ denotes the steering vector component of this element for the i -th sound source, $s_i(t)$ is the i -th source signal observed at the coordinate origin, and $n_{nm}(t)$ represents additive noise. The array received signal at time t is expressed as follows:

$$\mathbf{X}(t) = \mathbf{A}(\theta, \varphi) \mathbf{S}(t) + \mathbf{N}(t) = [x_{11}(t) \ x_{12}(t) \ \dots \ x_{NM}(t)]^T \quad (4)$$

where $\mathbf{A}(\theta, \varphi) = [\mathbf{a}(\theta_1, \varphi_1), \mathbf{a}(\theta_2, \varphi_2), \dots, \mathbf{a}(\theta_K, \varphi_K)]^T$ is an $MN \times K$ array manifold matrix, $\mathbf{S}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T$ is the received signals at the origin, and $\mathbf{N}(t) = [n_{11}(t), n_{12}(t), \dots, n_{NM}(t)]^T$ is the zero-mean Gaussian white noise uncorrelated with the signals with variance σ^2 .

The received signal model of the CHA is thus established. An examination of Eq. (2) reveals that the steering vector can be decomposed into two components. The first component, $\exp(-jk_0(n-1)h \cos \varphi)$, depends exclusively on the elevation angle φ . The second component, $\exp$

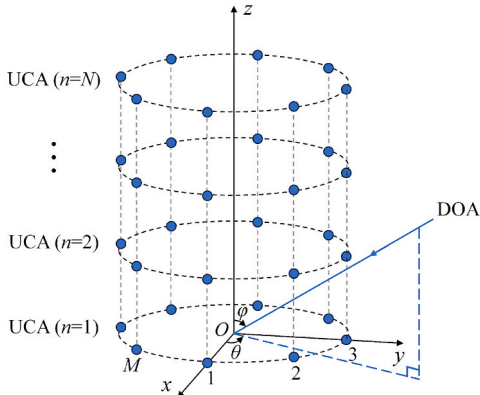


Fig. 1. Geometry of the CHA.

$(-jk_0 r \sin \varphi \cos(\theta - \gamma_m))$, is a function of both the azimuth and elevation angles. This structure implies that the DOA estimation can be performed in a decoupled manner: the elevation angles can be estimated first, independent of the azimuth, followed by the azimuth angle estimation. This sequential approach reduces computational complexity. Leveraging this angular separability, an improved DOA matrix method is proposed. Distinct from the classical DOA matrix method designed for two parallel uniform linear arrays [20], the proposed method extends the framework to CHAs. Furthermore, it incorporates spatial smoothing and denoising in elevation estimation stage to enhance robustness. A key advancement is the reformulation of azimuth estimation as a nonlinear least squares problem, which is solved iteratively to achieve high precision. The detailed implementation of the proposed method is elaborated in Section 2.2.

2.2. DOA matrix method for CHA

The proposed method leverages the axial translational invariance inherent in a CHA. A DOA matrix is first constructed based on this property. Subsequent eigenvalue decomposition of this matrix yields estimates for the elevation angles from its principal eigenvalues. The corresponding eigenvectors are then employed to estimate the azimuth angles. This process effectively decouples the estimation of the two angular parameters, leading to a substantial reduction in computational burden.

An N -layer CHA can be viewed as a vertical stack of N uniform circular arrays of identical radius. The received signal matrix of the n -th subarray is denoted as $\mathbf{X}_n(t) \in \mathbb{C}^{M \times L}$, where M is the number of elements per layer, L is the number of snapshots and $n = 1, 2, \dots, N$. According to Eq. (2), the array manifold matrices of two adjacent subarrays satisfy:

$$\mathbf{A}_n(\theta, \varphi) = \mathbf{A}_{n+1}(\theta, \varphi)\Phi \quad (5)$$

Here, Φ is a $K \times K$ diagonal matrix defined as:

$$\Phi = \text{diag}(e^{jk_0 h \cos \varphi_1}, e^{jk_0 h \cos \varphi_2}, \dots, e^{jk_0 h \cos \varphi_K}) \quad (6)$$

This equation indicates that the manifold matrices for adjacent subarrays differ only by a complex phase factor related to the elevation angle φ_k , which is independent of the azimuth angle.

To enhance the robustness of covariance matrix estimation, spatial smoothing is applied to both the auto-covariance matrices of the individual subarrays and the cross-covariance matrices between adjacent subarrays [24]. The smoothed auto-covariance matrix is expressed as:

$$\mathbf{R}_x = \frac{1}{NL} \sum_{i=1}^N \mathbf{X}_i(t) \mathbf{X}_i^H(t) \quad (7)$$

The smoothed cross-covariance matrix is expressed as:

$$\mathbf{R}_y = \frac{1}{(N-1)L} \sum_{i=1}^{N-1} \mathbf{X}_{i+1}(t) \mathbf{X}_i^H(t) \quad (8)$$

Denoising the covariance matrix can enhance the DOA estimation accuracy. Under the assumption of zero-mean, white Gaussian noise that is mutually independent across array elements, the noise expectation in the cross-covariance matrix is zero. Therefore, denoising is applied exclusively to the auto-covariance matrices. Performing eigenvalue decomposition on the smoothed autocovariance matrix $\mathbf{R}_x$ yields $(M-K)$ small eigenvalues η_i which are predominantly attributed to noise. The noise power is then estimated based on these eigenvalues as:

$$\sigma^2 = \frac{1}{M-K} \sum_{i=K+1}^M \eta_i \quad (9)$$

The denoised auto-covariance matrix is obtained as follows:

$$\mathbf{R}_{x0} = \mathbf{R}_x - \sigma^2 \mathbf{I}_M \quad (10)$$

An eigenvalue decomposition is performed on $\mathbf{R}_{x0}$, yielding K principal eigenvalues μ_i and their corresponding eigenvectors $\mathbf{V}_i$, $i = 1, \dots, K$. The pseudo-inverse matrix of $\mathbf{R}_{x0}$ is subsequently constructed from these

components as:

$$\mathbf{R}_{x0}^\# = \sum_{i=1}^K \mu_i^{-1} \mathbf{V}_i \mathbf{V}_i^H \quad (11)$$

Subsequently, the DOA matrix is constructed as:

$$\mathbf{R} = \mathbf{R}_y \mathbf{R}_{x0}^\# \quad (12)$$

Yin et al. [20] have proven that the DOA matrix satisfies the following relationship:

$$\mathbf{R} \mathbf{A}_1 = \mathbf{A}_1 \Phi \quad (13)$$

Eq. (13) indicates that the K non-zero eigenvalues of the DOA matrix $\mathbf{R}$ are precisely the diagonal elements of Φ , and the subspace spanned by their corresponding eigenvectors coincides with that spanned by the array manifold $\mathbf{A}_1$ of the first layer. It should be noted that the relationship in Eq. (13) strictly holds for theoretical covariance matrices defined by statistical expectations, as established in the original DOA matrix method [20]. In practice, the sample covariance matrices used in Eqs. (7)-(12) serve as statistical estimators of these theoretical values. Spatial smoothing maintains the translational invariance structure, while the denoising step eliminates noise contributions from the auto-covariance matrix. Consequently, the DOA matrix constructed from these samples approximates the ideal relationship, with the approximation accuracy consistently improving as the number of snapshots increases. Therefore, performing an eigenvalue decomposition on $\mathbf{R}$ allows separate extraction of information related to elevation and azimuth, while ensuring automatic parameter pairing.

Denote the K non-zero eigenvalues of $\mathbf{R}$ as ξ_i and their corresponding eigenvectors as $\mathbf{u}_i$, $i = 1, 2, \dots, K$. From Eq. (6), the relationship $\xi_i = \exp(jk_0 h \cos \varphi_i)$ is established. Consequently, the elevation angle can be estimated via the following equation:

$$\hat{\varphi}_i = \arccos\left(\frac{\angle \xi_i}{k_0 h}\right) \quad (14)$$

where $\angle \xi_i$ represents the principal phase of eigenvalue ξ_i . It is worth noting that the uniqueness of the elevation estimation depends on the inter-layer spacing h . Although the spatial Nyquist criterion ($h \leq \lambda/2$) guarantees unambiguous estimation over the full range $\varphi \in [0, \pi]$, spacings exceeding this limit remain viable provided that the elevation angles are confined to a specific region. For the array configuration used in this work, each circular layer has $M = 8$ elements with adjacent-element spacing of $\lambda/2$, yielding a cylinder radius of $r = 0.65\lambda$. The vertical inter-layer spacing is set equal to the radius, i.e., $h = 0.65\lambda$. Consequently, the unambiguous elevation range is approximately $\varphi \in [40^\circ, 140^\circ]$. All simulations and experiments presented herein are conducted within this valid region to avoid spatial ambiguity.

For azimuth estimation, the eigenvector $\mathbf{u}_i$ and the steering vector $\mathbf{a}_1(\theta_i, \varphi_i)$ of the first-layer UCA are related by a complex scalar factor α_i , such that $\mathbf{u}_i = \alpha_i \mathbf{a}_1(\theta_i, \varphi_i)$. By normalizing both $\mathbf{u}_i$ and $\mathbf{a}_1(\theta_i, \varphi_i)$ with respect to their first element, the scalar factor α_i can be eliminated, yielding:

$$\tilde{\mathbf{u}}_i = \frac{\mathbf{u}_i}{[\mathbf{u}_i]_1}, \tilde{\mathbf{a}}_1(\theta) = \frac{\mathbf{a}_1(\theta, \hat{\varphi}_i)}{[\mathbf{a}_1(\theta, \hat{\varphi}_i)]_1} \quad (15)$$

where $[\bullet]_1$ denotes the first element of the vector.

However, the UCA steering vector lacks a Vandermonde structure, resulting in a nonlinear relationship between its phase and azimuth angle. This precludes a closed-form solution and leads to the following nonlinear least squares problem:

$$\hat{\theta}_i = \argmin \|\tilde{\mathbf{u}}_i - \tilde{\mathbf{a}}_1(\theta)\|_2^2, \theta \in [-\pi, \pi] \quad (16)$$

It is noted that the phase of $\tilde{\mathbf{u}}_i$, derived from eigen-decomposition, is naturally wrapped into $[-\pi, \pi]$, whereas the theoretical phase of $\tilde{\mathbf{a}}_1(\theta)$, given by $k_0 r \sin(\hat{\varphi}_i) [\cos(\theta - \gamma_m) - \cos(\theta)]$, may fall outside this interval. To avoid phase wrapping, the phase difference is reduced to $[-\pi, \pi]$ via a

modular operation, which is equivalent to computing the shortest arc length on the unit circle. This ensures continuity of the cost function and prevents the optimizer from being misled by artificial 2π jumps. In addition, coarse grid-search initialization ensures the starting estimate is already close to the true value, keeping phase differences small throughout the iteration and further enhancing robustness.

The Gauss-Newton method is adopted to solve this nonlinear least squares (NLS) problem efficiently, with a coarse grid-search whose step size is 10° providing the initial estimate. By utilizing the first-order derivative information of the steering vector with respect to the azimuth angle, it achieves rapid convergence, typically with a few iterations. This method overcomes the efficiency limitation in traditional grid-search methods.

The implementation of the proposed method proceeds through the following steps:

Step 1: Compute the auto-covariance matrices of each subarray and the cross-covariance matrices between adjacent subarrays. Apply spatial smoothing to these matrices to obtain the smoothed covariance matrices, denoted as $\mathbf{R}_x$ and $\mathbf{R}_y$.

Step 2: Perform eigenvalue decomposition on $\mathbf{R}_x$. Estimate the noise power from the smallest $(M-K)$ eigenvalues, and use this estimate to obtain the denoised covariance matrix $\mathbf{R}_{x0}$.

Step 3: Construct the DOA matrix $\mathbf{R}$ according to Eq. (12) and perform eigenvalue decomposition on it.

Step 4: Estimate the elevation angle $\hat{\varphi}_i$ using Eq. (14).

Step 5: Initialize the solver with the coarse grid-search results. Solve the nonlinear least squares problem in Eq. (16) via Gauss-Newton method to obtain the estimated azimuth angle $\hat{\theta}$.

2.3. Measurement uncertainty analysis

The proposed method constitutes an indirect measurement process in which the measurands θ and φ are derived from the array signal data through sequential algebraic operations. The standard measurement uncertainty is evaluated by propagating uncertainty through each functional stage.

2.3.1. Elevation uncertainty

The elevation angle is determined by the functional relationship in Eq. (14), which depends on three input quantities: the eigenvalue phase $\angle \xi_i$, the wavenumber $k_0 = 2\pi f/c$ and the inter-layer spacing h . Since the signal frequency f is controlled by a stable signal generator, its contribution to $u(k_0)$ is negligible, giving $u(k_0)/k_0 \approx u(c)/c$. Applying the law of propagation of uncertainty yields the combined standard elevation uncertainty:

$$u(\hat{\varphi}_i) = \sqrt{\frac{u^2(\angle \xi_i)}{k_0^2 h^2 \sin^2 \varphi_i} + \cot^2 \varphi_i \left[\frac{u^2(c)}{c^2} + \frac{u^2(h)}{h^2} \right]} \quad (17)$$

Under the stochastic signal model with L snapshots, first-order matrix perturbation analysis of the DOA matrix eigenvalue yields $u(\angle \xi_i) \approx 1/\sqrt{(N-1) \cdot L \cdot \rho_i}$, where ρ_i is the SNR of the i -th source, giving the full expression:

$$u(\hat{\varphi}_i) = \sqrt{\frac{1}{k_0^2 h^2 \sin^2 \varphi_i (N-1) L \rho_i} + \cot^2 \varphi_i \left[\frac{u^2(c)}{c^2} + \frac{u^2(h)}{h^2} \right]} \quad (18)$$

The two terms in Eq. (18) correspond to Type A and Type B uncertainty contributions, respectively. The Type A term diverges as $1/\sin(\varphi_i)$ near the array endfire direction ($\varphi_i \rightarrow 0^\circ$ or 180°), while the Type B term scales as $\cot(\varphi_i)$ and vanishes at broadside ($\varphi_i = 90^\circ$). Consequently, in the vicinity of broadside the elevation measurement is dominated entirely by stochastic noise and is insensitive to sound speed and inter-layer spacing errors. Both terms diverge near endfire, confirming that the accuracy degradation observed experimentally at large off-broadside angles is an inherent geometric constraint common to all axial

translational invariance-based array measurement systems, irrespective of the method employed.

2.3.2. Azimuth uncertainty

The azimuth angle is estimated by solving the NLS problem in Eq. (16) via the Gauss-Newton method. Assuming the phase residuals $\mathbf{e}(\theta) = \angle \tilde{\mathbf{u}}_i - \angle \tilde{\mathbf{a}}_1(\theta)$ are i.i.d with zero mean and variance σ_e^2 , the asymptotic NLS estimator variance is $\text{var}(\hat{\theta}_i) = \sigma_e^2 / \|\mathbf{J}_\theta\|^2$. For a UCA with M elements and radius r , the Jacobian norm evaluates to $\|\mathbf{J}_\theta\|^2 = (k_0 r \sin \hat{\varphi}_i)^2 \cdot M/2$ via the identity $\sum_{m=0}^{M-1} \sin^2(\theta - \gamma_m) = M/2$. The phase residual standard deviation is estimated from eigenvector perturbation analysis as $\sigma_e \approx 1/\sqrt{M(N-1)L\rho_i}$, yielding the Type A standard uncertainty:

$$u_A(\hat{\theta}_i) \approx \frac{\sqrt{2}}{k_0 r \sin \hat{\varphi}_i M \sqrt{(N-1)L\rho_i}} \quad (19)$$

In addition to the Type A term, elevation estimation error propagates into $\hat{\theta}_i$ with cross-sensitivity $|\partial \hat{\theta}_i / \partial \hat{\varphi}_i| \leq \cot \hat{\varphi}_i$, and errors in sound speed c and array radius r contribute a Type B term bounded by $\sqrt{u^2(c)/c^2 + u^2(r)/r^2}$. The combined standard azimuth uncertainty is therefore:

$$u_c(\hat{\theta}_i) \approx \sqrt{\frac{2}{k_0^2 r^2 \sin^2 \hat{\varphi}_i \cdot M^2 (N-1) L \rho_i} + \cot^2 \hat{\varphi}_i \cdot u^2(\hat{\varphi}_i) + \frac{u^2(c)}{c^2} + \frac{u^2(r)}{r^2}} \quad (20)$$

Eq. (20) shares the same $1/\sin \hat{\varphi}_i$ singularity as the Type A term in Eq. (18), confirming a common endfire degradation mechanism for both angle estimates. Near broadside the coupling term vanishes ($\cot \hat{\varphi}_i \approx 0$) and is further suppressed by the covariance denoising in Step 2. A quantitative uncertainty budget based on Eqs. (18) and (20) is presented in Section 5.4.

3. Simulations

This section presents a systematic metrological evaluation of the proposed method through numerical simulations. For comparison, two classical DOA estimation methods are included: MUSIC [15] and TLS-ESPRIT [19]. Root-MUSIC is not adopted as a baseline because the CHA azimuth manifold lacks the Vandermonde structure required for polynomial rooting, and its beamspace variant suffers from phase-mode truncation error for the element count used here ($M = 8$), as discussed in the Introduction [17,18] and [25]. The simulation scenarios employ a CHA with $N = 4$ layers and $M = 8$ elements per layer, resulting in a total of 32 elements. The inter-element spacing within each circular layer is set to $\lambda/2$, which determines a cylinder radius of $r = 0.65\lambda$. The vertical spacing between adjacent layers is also set to 0.65λ . It is assumed that K far-field, narrowband, uncorrelated sources of equal power impinge on the array. The number of sources K is assumed to be known.

Six metrological metrics are adopted to characterize the measurement performance of the proposed method:

(1) Estimation bias characterizes the systematic measurement error of the method:

$$\text{Bias} = \frac{1}{QK} \sum_{i=1}^Q \sum_{k=1}^K (\hat{\alpha}_k^{(i)} - \alpha_k) \quad (21)$$

where Q denotes the number of Monte Carlo trials, α_k is the true angle of the k -th source (azimuth angle θ_k and elevation angle φ_k) and $\hat{\alpha}_k^{(i)}$ is its estimate in the i -th trial. All simulation results in this section are based on $Q = 500$ independent Monte Carlo trials. A bias statistically indistinguishable from zero indicates that the method is unbiased under the simulated conditions.

- (2) Root mean square error (RMSE) combines both systematic and random errors in DOA estimates. It is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{QK} \sum_{i=1}^Q \sum_{k=1}^K (\hat{\alpha}_k^{(i)} - \alpha_k)^2} \quad (22)$$

Using the bias-variance decomposition, $\text{RMSE}^2 = \text{Bias}^2 + \text{Var}(\hat{\alpha})$, where $\text{Var}(\hat{\alpha})$ is the variance of the estimates. If the method is unbiased, the RMSE reduces to the standard deviation of the estimates, which can be used as an A-type standard uncertainty component.

- (3) Cramér-Rao Bound (CRB) establishes the fundamental metrological limit of the DOA measurement, providing the minimum achievable variance of any unbiased measurement method of the measurand under the given operating conditions. For the narrowband far-field DOA estimation scenario under the deterministic signal model, the CRB matrix for the parameters vector $\boldsymbol{\eta} = [\theta, \varphi]^T$ is given by:

$$\text{CRB}(\boldsymbol{\eta}) = \frac{1}{2L \cdot \text{SNR}} [\text{Re}(\mathbf{D}^H \boldsymbol{\Pi}_A^\perp \mathbf{D})]^{-1} \quad (23)$$

where L denotes the number of snapshots, SNR is the signal-to-noise ratio, $\mathbf{D} = [\partial \mathbf{a} / \partial \theta, \partial \mathbf{a} / \partial \varphi]$ is the derivative matrix of the steering vector $\mathbf{a}$ with respect to the DOA parameters $\boldsymbol{\eta} = [\theta, \varphi]^T$, and $\boldsymbol{\Pi}_A^\perp = \mathbf{I} - \mathbf{a}(\mathbf{a}^H \mathbf{a})^{-1} \mathbf{a}^H$ is the orthogonal projection matrix onto the null space of the steering vector $\mathbf{a}$. The square roots of the diagonal entries of $\text{CRB}(\boldsymbol{\eta})$ give the theoretical lower bounds of RMSE for azimuth and elevation, respectively. CRB curves are superimposed on the RMSE figures as the reference baseline.

- (4) The expanded measurement uncertainty with coverage factor $k = 2$ yields an approximately 95% coverage interval for the DOA measurand:

$$U_{95} = 2\hat{\sigma}_{MC} \quad (24)$$

where $\hat{\sigma}_{MC}$ is the sample standard deviation of the Q measurement results at each operating condition, providing an empirical estimate of the combined standard measurement uncertainty u_c . The coverage interval $[\hat{\alpha} - U_{95}, \hat{\alpha} + U_{95}]$ represents the range within which the true value of the measurand is expected to lie with approximately 95% probability for a single measurement realization.

- (5) Measurement resolution: the resolution limit of the DOA estimation is characterized by the probability of successful resolution (PSR). For a two sources case, a trial is considered successfully resolved if the measurement error for each source is smaller than half of their angular separation. The PSR is calculated as the ratio of successfully resolved trials to the total number of Monte Carlo trials.
- (6) Measurement latency: average running time is used to serve as an indicator of the measurement latency, which determines its suitability for real-time acoustic tracking applications.

In the following subsections, the metrological performance of the proposed method is evaluated from three aspects: estimation accuracy, measurement resolution, and measurement latency.

3.1. Measurement accuracy evaluation

First, the DOA estimation accuracy of the proposed method is

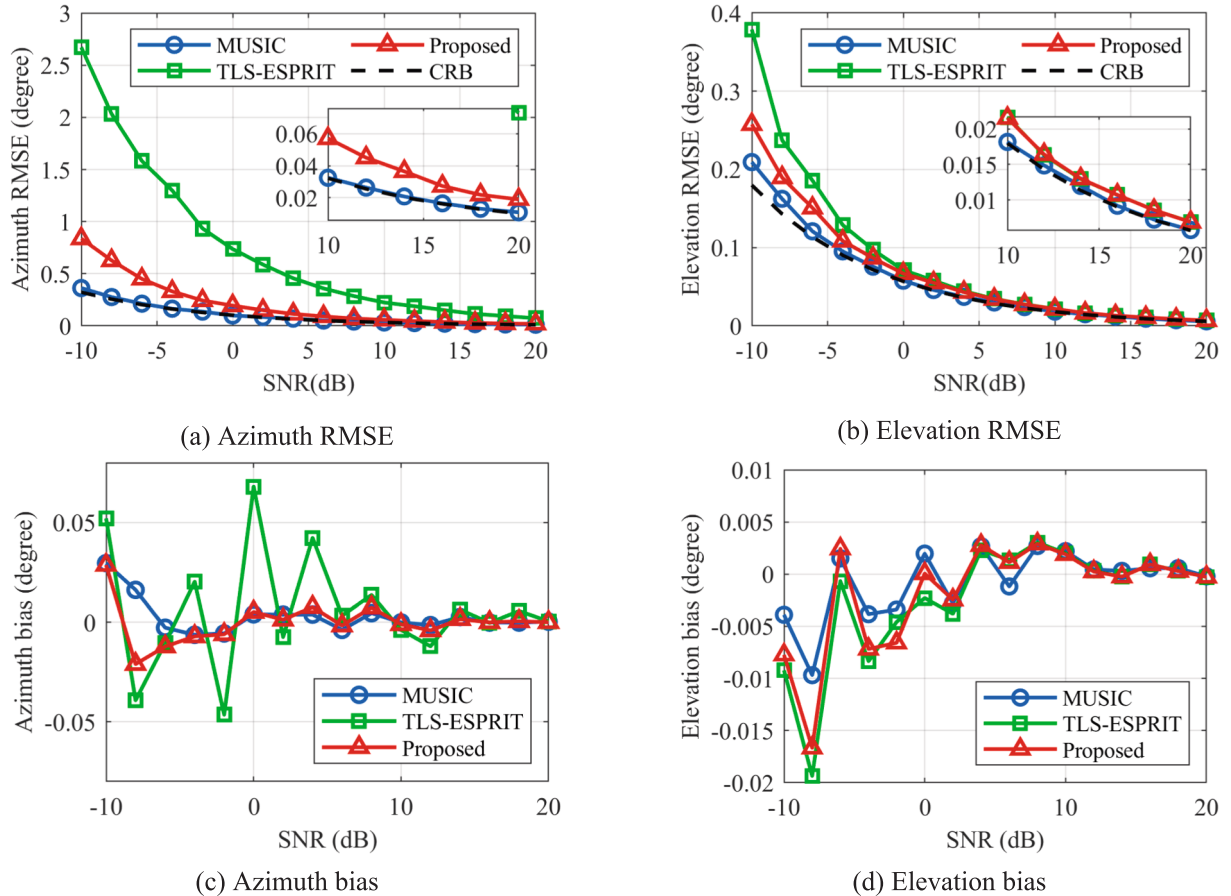


Fig. 2. Estimation RMSEs and biases of azimuth and elevation angles with different SNRs.

compared with that of MUSIC and TLS-ESPRIT under different signal-to-noise ratio (SNR) conditions. The simulation scenario is configured as follows: a single far-field, narrowband source impinges from $(\theta_1, \varphi_1) = (60^\circ, 50^\circ)$, with the number of snapshots fixed at 1000. The SNR ranges from -10 dB to 20 dB in steps of 2 dB. For each SNR level, 500 Monte Carlo trials are conducted. The RMSEs for both azimuth and elevation estimates are computed for all three methods, along with the CRB derived from $\text{CRB}(\eta)$ as the theoretical lower bound, as shown in Fig. 2 (a) and (b). As the SNR increases, the RMSE for all three methods decreases. The proposed method's RMSE approaches the CRB when $\text{SNR} \geq 5$ dB, indicating high statistical efficiency. In the low-SNR regime, MUSIC achieves the lowest RMSE, signifying the highest accuracy. The proposed method ranks second, yet consistently outperforms TLS-ESPRIT. This performance advantage stems from the proposed method's enhanced robustness in low-SNR environments. This robustness is attributed to two key features: the acquisition of a more reliable signal subspace estimation via spatial smoothing and covariance matrix denoising and the formulation of azimuth estimation as a nonlinear optimization problem. However, the proposed method is fundamentally a parametric approach. Consequently, any estimation error present in the initial signal subspace may propagate and accumulate through subsequent processing stages. This characteristic explains why its overall estimation accuracy, while excellent, remains marginally inferior to that of MUSIC. Fig. 2(c) and (d) present the azimuth and elevation estimation biases of the three methods, respectively. The biases of all three methods remain close to zero across the entire SNR range, indicating that all estimators are effectively unbiased under the simulated conditions.

In addition to SNR, the number of snapshots is another critical factor affecting DOA estimation accuracy. To investigate its impact, simulations are conducted with the SNR fixed at 10 dB while the number of snapshots varies from 100 to $10,000$. All other parameters remain consistent with the previous settings. Fig. 3(a) and (b) show the RMSEs of azimuth and elevation angle estimates with different numbers of snapshots, along with the CRB baseline. As the number of snapshots

increases, RMSEs of all three methods decrease. The proposed method's RMSE approaches the CRB when snapshots exceed 1000. For azimuth estimation, MUSIC maintains the lowest RMSE, followed by the proposed method, which surpasses TLS-ESPRIT. For elevation estimation, the performance of the three methods is comparable when the number of snapshots is limited. However, as the number of snapshots increases, the RMSE of MUSIC declines sharply. In contrast, the RMSE curves of the proposed method and TLS-ESPRIT nearly overlap and both are inferior to that of MUSIC. Fig. 3(c) and (d) show the estimation biases. All remain near zero across the snapshot range, confirming unbiasedness.

To further quantify the measurement reliability, the expanded uncertainty U_{95} with coverage factor $k = 2$ is derived from the sample standard deviation of the 500 Monte Carlo trials. Table 1 reports the U_{95} values for the proposed method under representative conditions.

3.2. Measurement resolution evaluation

This subsection evaluates the measurement resolution of the proposed method and compares it with MUSIC and TLS-ESPRIT in a dual-source scenario. Two far-field, narrowband sources of equal power are considered: the first fixed at $(\theta_1, \varphi_1) = (53^\circ, 45^\circ)$, the second at $(\theta_2, \varphi_2) = (53^\circ + \Delta, 45^\circ + \Delta)$, with the angle separation Δ increasing from 2° to 20° . The SNR is fixed at 10 dB and the number of snapshots is 1000. In this subsection, MUSIC employs a 0.1° search grid in both azimuth and

Table 1

Expanded uncertainty (U_{95} , $k = 2$) of the proposed method under representative conditions.

Condition	Azimuth U_{95} (degree)	Elevation U_{95} (degree)
SNR = 0 dB, $L = 1000$	± 0.39	± 0.13
SNR = 10 dB, $L = 1000$	± 0.11	± 0.04
SNR = 20 dB, $L = 1000$	± 0.04	± 0.01
SNR = 10 dB, $L = 500$	± 0.17	± 0.04
SNR = 10 dB, $L = 10000$	± 0.04	± 0.01

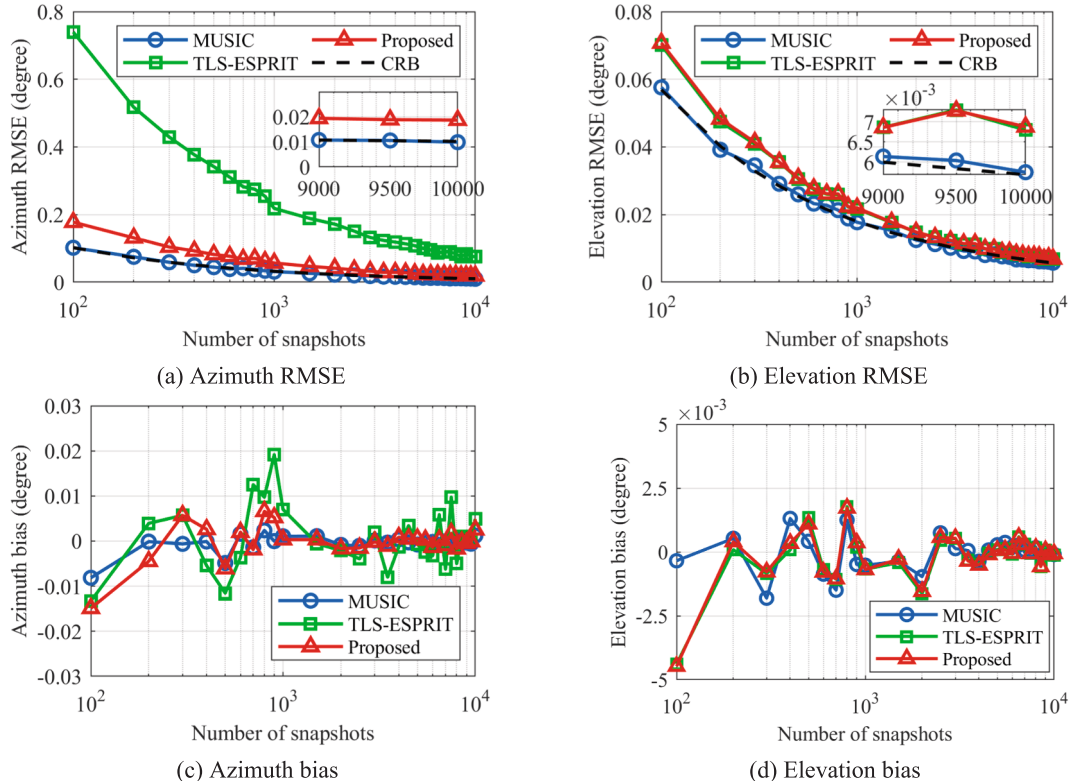


Fig. 3. Estimation RMSEs and biases of azimuth and elevation angles with different numbers of snapshots.

elevation to ensure that angular quantization does not limit the resolution assessment.

Fig. 4(a) plots the probability of successful resolution (PSR) with different angle separations. The PSR for the three methods improves as the angle separation widens. At this snapshot level, the resolution advantage of the proposed method can be attributed to more efficient covariance estimation. MUSIC relies on a single 32×32 full-array covariance matrix, whose reliable estimation requires a snapshot count proportional to its dimension. The proposed method instead builds the DOA matrix from 8×8 layer-wise covariance matrices: $N = 4$ auto-covariance matrices from Eq. (7) and $N - 1 = 3$ adjacent-layer cross-covariance matrices from Eq. (8). Their smaller dimension yields a much higher snapshot-to-dimension ratio at any given L . Spatial smoothing across the $N = 4$ subarrays further improves reliability by averaging estimates from distinct array layers, and the eigenvalue-based denoising in Eq. (10) removes the noise bias of the auto-covariance matrices. The resulting DOA matrix therefore encodes true angular information more faithfully under limited snapshots, giving the proposed method the lowest resolution threshold. Specifically, it attains 100% PSR at $\Delta \approx 3^\circ$, whereas MUSIC and TLS-ESPRIT require approximately 4° and 12° , respectively.

To further characterize this advantage, Fig. 4(b) shows the PSR as L is swept from 10^2 to 10^5 at a fixed $\Delta = 2^\circ$, with SNR = 10 dB and $Q = 500$ Monte Carlo trials. The PSR gap between the proposed method and MUSIC reaches a peak of approximately 0.33 at $L = 10^3$, subsequently narrowing to essentially zero at $L = 10^5$, where both achieve near-unity PSR. Once L is large enough to estimate the 32×32 covariance matrix reliably, the finite-sample advantage vanishes. In practical real-time underwater tracking, the available snapshot count is often limited by target motion and channel non-stationarity, a regime in which the proposed method provides a measurable resolution advantage under the tested conditions.

3.3. Measurement latency evaluation

This subsection analyzes the measurement latency of the proposed method by comparing the average running time with that of other methods. The running time for a single DOA estimation is recorded for each method in the previous snapshot simulations and averaged over 500 Monte Carlo trials. For MUSIC, two search step sizes are considered: 0.1° and 1° . The 0.1° setting is used in Section 3.1 to represent the theoretical upper bound of MUSIC's accuracy, while the 1° setting is included here to reflect a practical engineering trade-off between accuracy and latency. Fig. 5 illustrates the average running time with different numbers of snapshots.

MUSIC with a 0.1° grid incurs the highest computational burden, averaging approximately 26 s. When the step size is relaxed to 1° , the running time decreases dramatically to approximately 0.35 s. In both

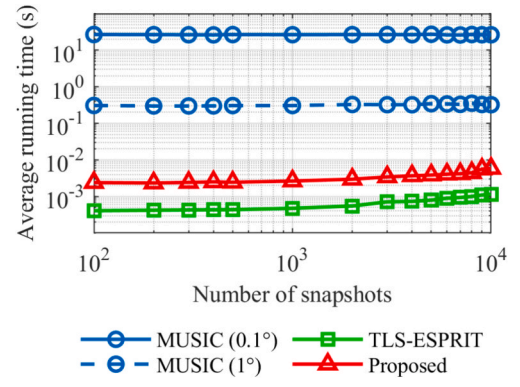


Fig. 5. Average running time with different numbers of snapshots.

configurations, the running time of MUSIC remains nearly constant regardless of the number of snapshots, indicating that the dominant cost originates from an exhaustive 2D spatial spectrum search. The computational complexity of this search is $O(N_{azi}N_{ele}(MN)^2)$, where N_{azi} and N_{ele} denote the numbers of azimuth and elevation search grid points, respectively. The substantial computation required under fine grid resolution hinders its suitability for real-time processing. In contrast, TLS-ESPRIT shows the shortest average running time and highest efficiency. Its complexity is $O((MN)^2L + (MN)^3)$, implying a linear dependence on the number of snapshots L , which aligns with the trend in Fig. 5. The high efficiency stems from its reliance on algebraic operations, avoiding iterative optimization or search operations. The proposed method's running time falls between that of TLS-ESPRIT and that of MUSIC with a 1° grid. It is roughly one order of magnitude higher than that of TLS-ESPRIT, yet significantly lower than that of MUSIC even at a coarse 1° grid setting, and at least three orders of magnitude lower than that of MUSIC with a 0.1° grid. This difference reflects the inherent cost of the 2D spectral search rather than any implementation-specific overhead. Its computational complexity is $O(NM^2L + M^3 + KM)$, indicating that the running time scales slowly with L . The additional computational overhead stems from the procedures introduced for enhanced robustness and accuracy, namely spatial smoothing, denoising, and nonlinear optimization for azimuth estimation.

In summary, the simulation results show a favorable accuracy-latency trade-off. The proposed method's azimuth accuracy is second only to MUSIC and well above TLS-ESPRIT, while its elevation accuracy remains comparable to the other methods under limited-snapshot conditions. Its running time is lower than that of MUSIC even with the coarse 1° grid and orders of magnitude lower than that of the fine-grid MUSIC. Compared to TLS-ESPRIT, it trades a small, millisecond-scale latency increase for markedly improved robustness and precision.

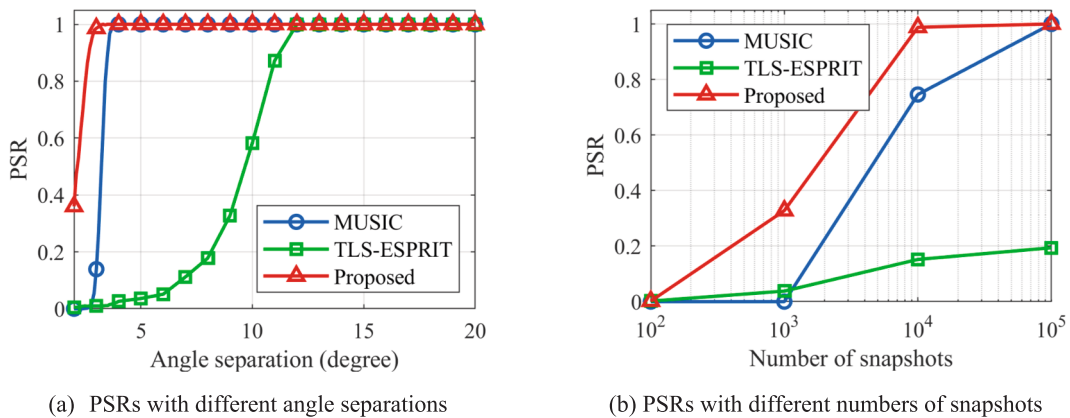


Fig. 4. Measurement resolution of the three methods.

3.4. Robustness to coherent sources

This subsection evaluates the robustness of the proposed method to coherent sources, a scenario relevant to underwater multipath propagation where reflected replicas of the same source produce coherent arrivals. Spatial smoothing, applied to both the auto-covariance and cross-covariance matrices during DOA matrix construction, restores the rank of the covariance matrix under coherent conditions.

A simulation is conducted with the coherence coefficient ρ swept from 0 (incoherent) to 1 (fully coherent). Two sources are placed at $(\theta_1, \varphi_1) = (53^\circ, 45^\circ)$ and $(\theta_2, \varphi_2) = (63^\circ, 55^\circ)$, respectively. The SNR is fixed at 10 dB and the number of snapshots is 1000. The proposed method is compared with an ablated variant that disables spatial smoothing, using only a single layer for covariance matrix estimation. As shown in Fig. 6(a), the proposed method maintains a PSR of 100% across the entire coherence range $\rho \in [0, 1]$. In contrast, the variant without spatial smoothing begins to fail at $\rho \approx 0.3$ and drops to near-zero PSR for $\rho \in [0.4, 1]$, confirming that spatial smoothing is essential for coherent scenarios. Fig. 6(b) presents the conditional RMSE and bias for successful trials. Both azimuth and elevation RMSE increase monotonically with ρ , exhibiting a pronounced asymmetry: azimuth RMSE rises gradually from approximately 0.13° at $\rho = 0$ to 1.48° at $\rho = 1$, whereas elevation RMSE remains near 0.01° for small ρ before rising sharply beyond $\rho \approx 0.85$ to reach 3.2° at $\rho = 1$. This difference stems from the geometric constraints of the cylindrical array. The limited vertical aperture reduces the effectiveness of spatial smoothing, making elevation estimation significantly more susceptible to coherence-induced rank deficiency than azimuth estimation, which benefits from the full horizontal aperture.

Regarding estimation bias, the azimuth bias remains near zero with a slight positive trend ($|\text{bias}| < 0.07^\circ$ for $\rho \leq 0.85$), reversing sign beyond $\rho \approx 0.85$ and reaching approximately -0.08° at $\rho = 1$. The elevation bias decreases monotonically from near 0° at $\rho = 0$ to approximately -0.29° at $\rho = 1$. This coherence-dependent bias indicates that spatial smoothing, while effective at partially restoring matrix rank, cannot fully maintain estimation consistency in the highly coherent regime ($\rho > 0.9$). Overall, the proposed method delivers reliable joint azimuth-elevation estimation for $\rho \leq 0.85$, beyond which both RMSE and bias deteriorate substantially, particularly in the elevation dimension.

4. Experiments

Experimental validation of the proposed method was performed in an anechoic tank. Leveraging a CHA, the estimation accuracy and computational efficiency of the proposed method were systematically evaluated for both stationary and moving sound sources, with performance compared with that of MUSIC and TLS-ESPRIT.

The experiments were carried out in the large anechoic tank at the

Tianjin Academy of Water Transport Engineering of the Ministry of Transport. The tank dimensions are $30 \text{ m} \times 12 \text{ m} \times 8 \text{ m}$ and it provides a sound absorption coefficient $\geq 99\%$ for acoustic waves above 3 kHz. The overall layout of the experimental system is shown in Fig. 7. The underwater acoustic system consists of a 32-element CHA and a ST-150 acoustic transducer as the sound source. A photograph of the CHA is presented in Fig. 8. The array has a radius of 0.2 m. Its 32 hydrophones are uniformly distributed across four layers along its surface, with an inter-layer spacing of 0.2 m. The array was calibrated using the single-source method [34], with the calibration matrix averaged over multiple rotations of the CHA to improve accuracy. In the experimental configuration (Fig. 7), the array is positioned at location (2). Its coordinate origin is submerged 2 m below the water surface, with z-axis oriented vertically upward. The acoustic source is initially placed at location (1) in Fig. 7, at a distance of 7.28 m from the center of the array and at a deployment depth denoted as h_T . It emits a 5 kHz single-frequency sinusoidal pulse signal. The received signals are synchronously acquired by an FPGA-based system and subsequently transferred to a host computer for offline processing.

A 2D DOA estimation experiment with a single stationary acoustic source was first conducted to systematically evaluate the method's performance for different incident directions. The frequency of the acoustic source was set to 5 kHz. As shown in Fig. 7(4), the source was sequentially positioned at 54 spatial grid points by a test carriage. These points were defined by a combination of 18 preset azimuth angles and 3 preset elevation angles. At each fixed location, three independent sets of array data, each comprising $L = 1000$ snapshots, were recorded for subsequent DOA estimation and analysis.

A second experiment involving a single moving source was performed to further assess the method's tracking capability and robustness in a low-speed dynamic scenario. The source, towed by a test carriage, moved along the predefined trajectory marked by arrow (3) in Fig. 7. During this motion, the source's azimuth changed from -1.42° to 29.85° while it was maintained at a constant depth of 2 m. The towing speed was controlled at 1174 mm/min and the source continuously emitted a 5 kHz sinusoid signal. The received data stream was segmented into consecutive 1-second frames, each comprising $L = 10^5$ snapshots. Each frame was processed independently by the algorithms to produce a 2D DOA estimate, thereby enabling the tracking of both azimuth and elevation angles for the moving source.

5. Results and discussion

5.1. DOA estimation of a stationary acoustic source

The RMSE results for DOA estimation of a stationary acoustic source are shown in Fig. 9. The left and right columns of the figure display the RMSE for azimuth and elevation estimates, respectively. For all tested

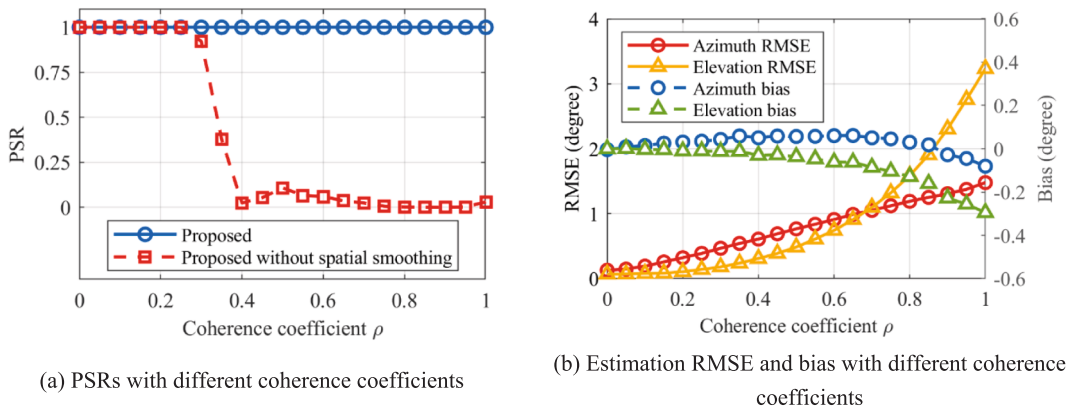


Fig. 6. Robustness to coherent sources.

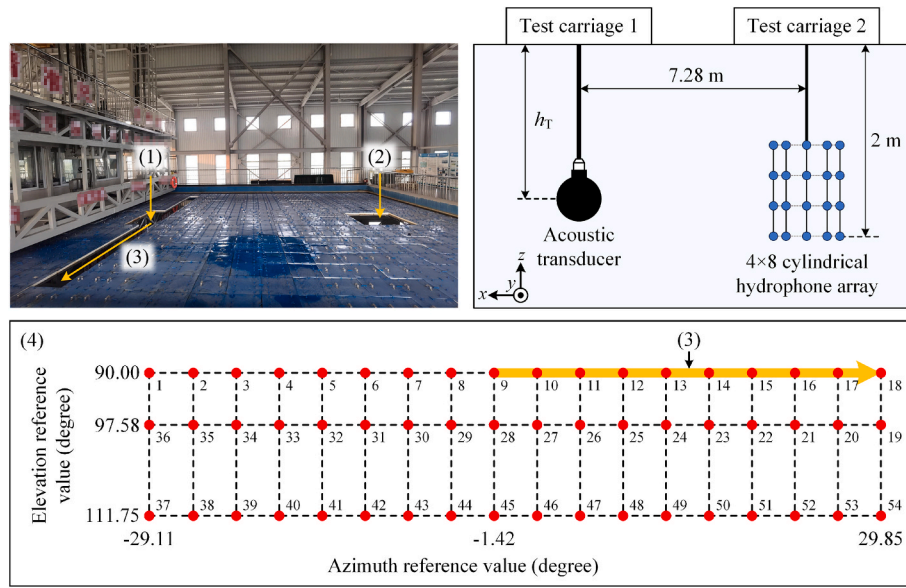


Fig. 7. Schematic of the experiment. (1) Initial position of the acoustic transducer, (2) position of the CHA, (3) trajectory of the source, and (4) grid of pre-defined azimuth and elevation test positions.

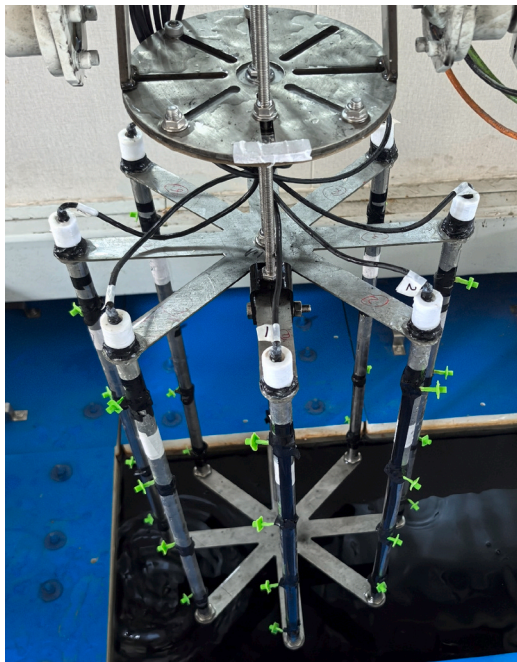


Fig. 8. Photograph of the developed CHA.

elevation angles, the proposed method achieves a lower azimuth RMSE than TLS-ESPRIT, while its performance is nearly comparable to, and occasionally surpasses, that of MUSIC at specific test points. When the source is near the array broadside direction, i.e., at elevation angles of 90° and 97.58° , the performance of the three methods is similar. A significant increase in elevation RMSE is observed for all three methods when the elevation angle increases to 111.75° , corresponding to the source deviating farther from the broadside. This performance degradation stems from the reduced effective vertical aperture of the CHA as the source moves off-broadside, which consequently lowers the angular resolution for elevation estimation.

To quantify the measurement repeatability of the proposed method, three independent data sets recorded at each position were processed separately. Four representative spatial positions were selected: three

positions sharing a common azimuth angle of 2.39° with elevation angles of 90° , 97.58° and 111.75° , respectively, and an additional point at $\theta = 29.85^\circ$, $\varphi = 90^\circ$. For each position, the sample standard deviation of the three azimuth and elevation estimates is computed as the Type A standard uncertainty $u_A(\theta) = s_\theta$ and $u_A(\varphi) = s_\varphi$, where s is the sample standard deviation of the three repeated estimates. The results are summarized in Table 2. All Type A standard uncertainties remain below 0.1° , demonstrating consistently high repeatability across the tested positions. These values confirm that random measurement variability contributes only a minor component to the overall measurement uncertainty. The experimentally determined u_A values at these representative positions are used as Type A inputs to the uncertainty budget presented in Section 5.4.

5.2. DOA estimation of a moving acoustic source

The DOA estimation results for the moving source are shown in Fig. 10 and Fig. 11. Fig. 10 compares the estimated trajectory of each method with the reference trajectory. Fig. 11 provides a quantitative performance assessment by plotting the absolute error (AE) over time, which includes the absolute azimuth error (AAE) and absolute elevation error (AEE). For azimuth tracking, both the proposed method and MUSIC closely follow the reference trajectory, with their AAE remaining within 4° throughout the motion. Conversely, the trajectory estimated by TLS-ESPRIT shows significant deviation from the reference, resulting in a larger AAE. For elevation tracking, the estimated trajectories and the corresponding AEE curves for the three methods are highly consistent. This indicates that their elevation estimation performance is similar, and all three are capable of dynamically tracking the low-speed moving targets in the elevation dimension.

5.3. Summary of experimental results

Table 3 summarizes the RMSE and average running time for each method under both stationary and moving source conditions. For azimuth estimation, MUSIC achieves the lowest RMSEs of 1.52° and 1.25° for stationary and moving sources, respectively. The proposed method follows closely at 1.90° and 2.25° , substantially outperforming TLS-ESPRIT with RMSEs of 4.72° and 5.21° . When MUSIC employs a coarser 1° grid, its running time decreases to approximately 350 ms, yet its RMSE remains nearly unchanged, indicating that the finer 0.1° grid

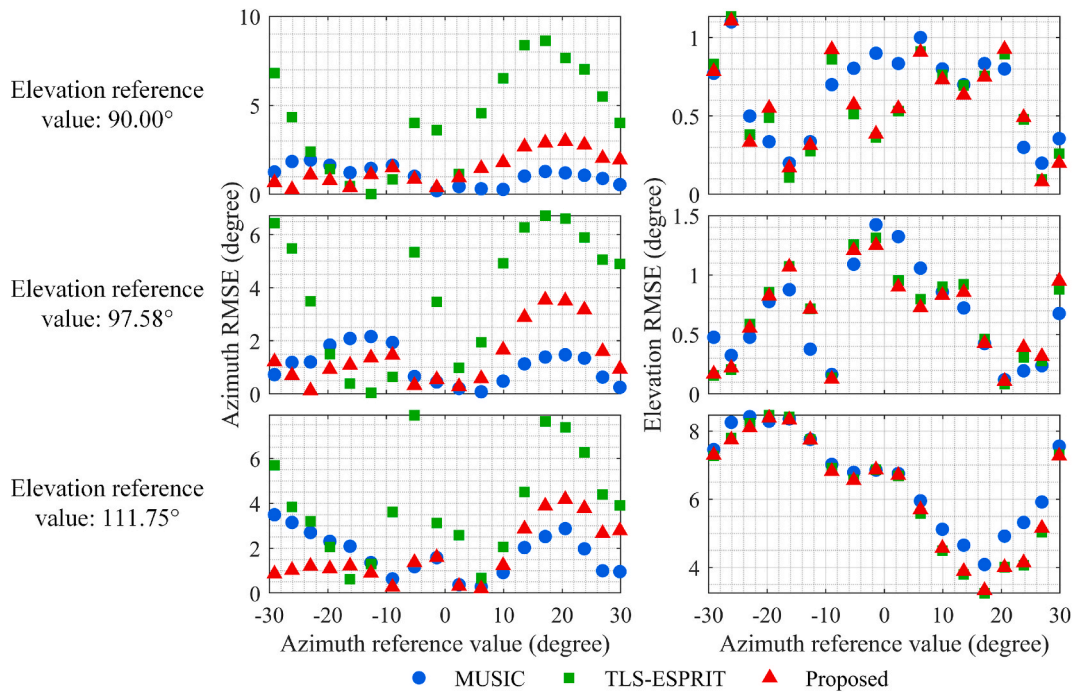


Fig. 9. Performance comparison of DOA estimation algorithms for a stationary acoustic source.

Table 2

Repeatability of DOA estimates at representative positions.

Position (θ, φ)	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	s_θ	$\hat{\varphi}_1$	$\hat{\varphi}_2$	$\hat{\varphi}_3$	s_φ
(2.39°, 90°)	3.31°	3.34°	3.37°	0.03°	90.53°	90.56°	90.55°	0.02°
(2.39°, 97.58°)	2.67°	2.62°	2.72°	0.05°	98.45°	98.46°	98.51°	0.03°
(2.39°, 111.75°)	2.73°	2.72°	2.65°	0.04°	105.05°	105.06°	105.04°	0.01°
(29.85°, 90°)	31.77°	31.86°	31.75°	0.06°	89.85°	89.72°	89.87°	0.08°

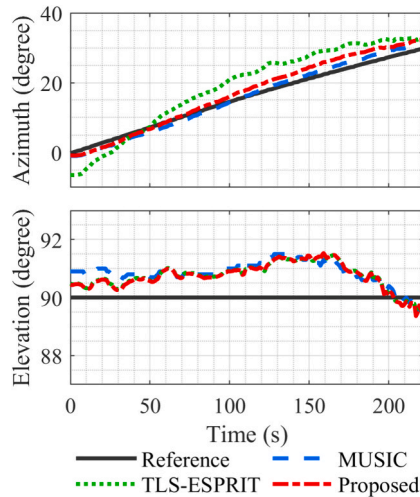


Fig. 10. Comparison of the estimated and reference trajectories for the moving acoustic source.

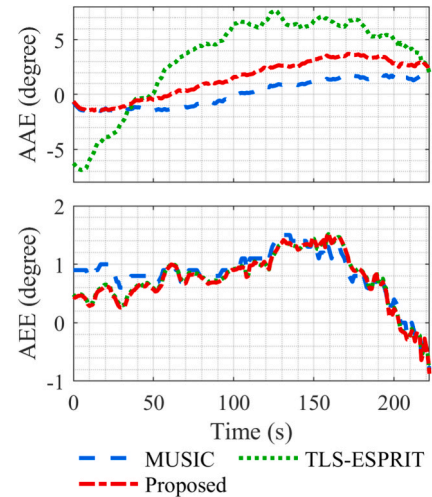


Fig. 11. Absolute error of DOA tracking for the moving acoustic source.

offers negligible accuracy improvements at a disproportionate computational cost. For elevation estimation, the proposed method and MUSIC perform comparably. The proposed method achieves marginally lower experimental RMSEs, exhibiting 3.77° versus 3.96° for the stationary source and 0.87° versus 0.93° for the moving source. In contrast, simulations indicate that MUSIC performs marginally better at high snapshot counts, as illustrated in Fig. 3(b). This sub-0.2° experimental

difference lies within the combined systematic uncertainty shared by all methods, as detailed in Table 4. Therefore, it does not constitute a meaningful elevation-accuracy advantage for either method. The accuracy comparison is dimension-dependent: MUSIC retains an advantage in azimuth, while the two methods are comparable in elevation. Regarding computational efficiency, both the proposed method and TLS-ESPRIT complete their calculations within milliseconds. Specifically, the proposed method runs approximately 77 times and 8 times

Table 3

Performance comparison of different methods for stationary and moving acoustic sources.

Condition	Methods	Azimuth RMSE (degree)	Elevation RMSE(degree)	Average Running Time (ms)
Stationary Source	MUSIC (0.1°)	1.52	3.96	26,550
	MUSIC (1°)	1.50	3.96	351.58
	TLS-ESPRIT	4.72	3.78	0.84
	Proposed	1.90	3.77	4.56
Moving Source	MUSIC (0.1°)	1.25	0.93	25,800
	MUSIC (1°)	1.30	0.97	350.48
	TLS-ESPRIT	5.21	0.87	4.76
	Proposed	2.25	0.87	46.20

Table 4

Uncertainty budget for DOA estimation at a representative position ($\theta = 2.39^\circ$, $\varphi = 90^\circ$).

Source	Type	Value	Distribution	Contribution u_i (azimuth)	Contribution u_i (elevation)
Repeatability (azimuth)	A	$s_\theta = 0.03^\circ$	Normal	0.03°	
Repeatability (elevation)	A	$s_\varphi = 0.02^\circ$	Normal		0.02°
Sound speed c	B	$u(c)/c = 0.5\%$	Rectangular	0.28°	0.00°
Inter-layer spacing h	B	$u(h) = 3$ mm	Rectangular		0.00°
Array radius r	B	$u(r) = 3$ mm	Rectangular	0.86°	0.00°
Residual calibration error	B	0.5°	Rectangular	0.50°	0.50°
Combined standard uncertainty u_c				1.03°	0.50°
Expanded uncertainty U_{95} ($k = 2$)				2.06°	1.00°

faster than the 1°-grid MUSIC for stationary and moving sources, respectively. The increased running time for the moving source is attributed to the snapshot-dependent complexity term, which scales with the number of snapshots in both the proposed method and TLS-ESPRIT, as detailed in Section 3.3. In summary, the proposed method strikes a favorable balance between measurement accuracy and computational efficiency, making it well-suited for latency-sensitive applications.

5.4. Discussion

5.4.1. Measurement uncertainty budget

Based on the uncertainty propagation model established in Section 2.3, a quantitative uncertainty budget is constructed for the proposed method. Table 4 summarizes the uncertainty components for the representative broadside position ($\theta = 2.39^\circ$, $\varphi = 90^\circ$), where the measurement repeatability is characterized by Type A standard uncertainties from Table 2. Type B uncertainty components include the sound speed c , the inter-layer spacing h , the array radius r , and residual calibration errors. The sound speed is estimated to be 1500 m/s with a relative standard uncertainty $u(c)/c = 0.5\%$ based on water temperature variations. The inter-layer spacing $h = 0.2$ m is measured with a ruler,

yielding $u(h) = 3$ mm. The array radius $r = 0.2$ m is subject to manual assembly tolerances, giving $u(r) = 3$ mm. These uncertainties are assumed to follow rectangular distributions and are converted to standard uncertainties accordingly. The residual calibration error, which encompasses unmodeled amplitude-phase mismatches after the single-source calibration procedure, is empirically estimated to be 0.5° .

At the selected broadside position, $\cot \varphi = 0$, which has distinct implications for each measurand. For the elevation angle, both Type B terms in Eq. (18) are multiplied by $\cot \varphi$ and therefore vanish. The combined standard elevation uncertainty $u_c(\varphi) = 0.5^\circ$ is thus governed entirely by the residual calibration error. For the azimuth angle, the $\cot \varphi = 0$ condition eliminates the cross-sensitivity coupling from elevation error into azimuth estimation. Nevertheless, $u(c)/c$ and $u(r)/r$ continue to contribute directly, yielding 0.28° and 0.86° , respectively. The array radius tolerance is the single largest contributor, accounting for approximately 70% of the total azimuth variance. This originates from manual assembly of the cylindrical frame, which introduces position deviations that produce angle-dependent phase distortions across the array manifold. The distortions cannot be fully corrected by the single-source calibration method in this study. The residual calibration error of 0.5° similarly reflects inaccuracies in positioning the auxiliary calibration source, which introduce a systematic bias that persists after compensation.

The scope of applicability of this budget warrants clarification. As the elevation departs from broadside, both the $\cot \varphi$ Type B terms and the $1/\sin \varphi$ Type A term in Eqs. (18) and (20) grow monotonically, amplifying the combined standard uncertainty. The budget in Table 4 therefore represents a lower bound on uncertainty within the valid elevation range $\varphi \in [40^\circ, 140^\circ]$, consistent with the pronounced RMSE increase observed at $\varphi = 111.75^\circ$ in Fig. 9. The expanded uncertainties $U_{95} = 2.06^\circ$ (azimuth) and $U_{95} = 1^\circ$ (elevation), each obtained with coverage factor $k = 2$, define 95% coverage intervals at the broadside position. The azimuth expanded uncertainty of 2.06° is consistent with the azimuth RMSE at the broadside as shown in Fig. 9, providing empirical validation of the budget. The elevation expanded uncertainty of 1° similarly matches the elevation RMSE at $\varphi = 90^\circ$ in Fig. 9, confirming the budget's accuracy at the optimal geometry. The aggregate elevation RMSE of 3.96° in Table 3, which includes off-broadside positions, is not directly comparable because the $\cot \varphi$ -driven Type B contributions are no longer negligible away from the broadside.

5.4.2. Performance in complex underwater environments and limitations

The preceding experimental results were obtained in a highly controlled anechoic tank environment. To assess the potential and limitations of the proposed method for practical applications, this subsection discusses its expected behavior in several representative complex underwater acoustic scenarios, including multipath propagation, reverberation, multiple sources, and open-water environments.

In underwater environments, multipath propagation generates coherent signal replicas that cause rank deficiency in the sample covariance matrix. The proposed method explicitly addresses this through spatial smoothing applied to both the auto-covariance and cross-covariance matrices during DOA matrix construction, which restores the rank of the covariance matrix by averaging over the four axially translated UCA layers. As demonstrated in Section 3.4, the method maintains a 100% probability of successful resolution across all coherence levels and achieves reliable joint azimuth–elevation estimation for coherence coefficients $\rho \leq 0.85$. Beyond this threshold, elevation estimation accuracy deteriorates markedly due to the limited vertical aperture of the CHA, which constrains the effectiveness of spatial smoothing in the axial direction relative to the azimuthal direction. A straightforward hardware-level mitigation is to increase the number of axially translated UCA layers, which extends the array aperture and provides more subarrays for spatial smoothing, thereby improving rank restoration at $\rho > 0.85$ without any modification to the proposed method.

Reverberation is physically distinct from multipath in that it raises the noise floor with a spatially correlated, non-white field. The denoising step in the proposed method relies on the spatially white noise assumption to estimate noise power from the smaller eigenvalues of the auto-covariance matrix. Under reverberant conditions, this assumption is violated and the noise power estimate becomes biased, consequently distorting the DOA matrix. The current method therefore does not inherently handle strong reverberation, and noise whitening or explicit reverberation modeling are identified as necessary extensions for future work. One feasible extension within the existing framework is to pre-whiten the received data using a noise covariance matrix estimated during a source-silent interval, leaving the DOA matrix construction and subsequent processing steps unchanged.

Regarding multi-source capacity, spatial smoothing reduces the effective covariance matrix dimension from $MN \times MN$ to $M \times M$, limiting the method to at most $K \leq M - 1 = 7$ simultaneous sources. This capacity-rank-restoration trade-off is inherent to any spatial smoothing-based approach. Within this capacity, under the limited-snapshot conditions evaluated in Section 3.2, the proposed method demonstrates superior two-source resolution compared with MUSIC and TLS-ESPRIT. A direct hardware-level extension is to increase the number of elements per UCA to $M' > M$, which raises the source capacity ceiling to $M'-1$ while leaving the proposed method otherwise unchanged. The resolution and coherent-source advantages reported here are established through rigorous simulations. Anechoic-tank experiments independently validate the single-source metrology and uncertainty budget. Multi-source experimental validation, which requires simultaneous acoustic sources, is deferred to future work due to hardware limitations.

In real open-water environments, several additional factors degrade performance. Depth-dependent sound speed profiles cause refraction and wavefront curvature, producing array manifold mismatch that is not compensated by the calibration procedure. Broadband sources would also violate the narrowband far-field model assumed in Section 2.1. Furthermore, temporal variations due to tides, internal waves, and turbulence render the covariance matrix non-stationary, reducing the effective number of snapshots available for reliable DOA matrix construction. The current method assumes a stationary, homogeneous medium and does not incorporate adaptive tracking mechanisms. These challenges can be addressed through incremental front-end modules: periodic steering vector recalibration using environmental sound speed profiles, narrowband filter bank decomposition for broadband sources and sliding-window covariance estimation for non-stationary channels. Each of these leaves the core DOA matrix construction and subsequent processing steps unchanged. Validation in open-water conditions and the development of appropriate compensation strategies represent the primary directions for future work.

6. Conclusion

A DOA matrix-based measurement method for 2D DOA estimation with CHAs is proposed. By decoupling elevation and azimuth estimation via axial translational invariance and solving for azimuth via Gauss-Newton iteration, the proposed method achieves elevation accuracy comparable to MUSIC, conceding only a modest margin in azimuth accuracy. Furthermore, it reduces computational complexity by up to two orders of magnitude compared to MUSIC with a 1° grid. In addition, it significantly outperforms TLS-ESPRIT in azimuth accuracy while matching its elevation performance, with only a modest increase in measurement latency. Simulations further demonstrate that the proposed method achieves a lower resolution threshold than MUSIC under limited-snapshot conditions, while maintaining effectiveness for coherent sources. To fully characterize this performance, a comprehensive metrological framework encompassing bias, RMSE, the CRB, expanded uncertainty, resolution, and latency is established. Experimental results from an anechoic tank confirm millisecond-order latency, an azimuth RMSE below 3° for both stationary and moving sources, and

elevation RMSE below 1° for moving sources. As these results were obtained under ideal conditions, the method's robustness in realistic underwater environments, characterized by multipath, reverberation and non-stationary noise, requires further validation. Future work will therefore focus on open-water trials to assess and enhance its practical engineering applicability.

7. Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the authors used DeepSeek to improve the language and readability of the manuscript. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

CRediT authorship contribution statement

Xinjing Huang: Writing – review & editing, Supervision, Funding acquisition, Conceptualization, Validation, Resources. **Hangyang Chen:** Writing – original draft, Methodology, Data curation, Conceptualization, Validation, Visualization. **Chenwu Jia:** Software, Data curation, Investigation, Writing – review & editing. **Zan Li:** Data curation, Writing – review & editing, Validation, Formal analysis. **Jinyu Ma:** Writing – review & editing, Supervision, Resources, Project administration, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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